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Decision Making Framework in Production, Repairing and Replacement Machine Using Dynamic Programming and Bayesian Inference

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Abstract

The management of machine replacement is an effective decision-making process in controlling disruptions in industries. Specifically, determining a proper policy for replacing equipment and machines can decrease production costs. Therefore, it is of great importance for decision-makers to take an appropriate machine replacement policy into account. Hence, in this study, a maintenance policy is proposed which includes two costs: (i) preventive maintenance and (ii) quality control. The optimal policy is determined using both Bayesian inference and dynamic programming approaches. Specifically, a lifetime function and its parameters are modeled using Bayes' theorem. In addition, a dynamic programming model is applied to determine the best decision-making policy among three policies: (i) replacing the machine, (ii) continuing the process, and (iii) repairing the machine. Also, a numerical example is carried out, and some discussions are provided.

Keywords: Bayesian inference, Dynamic programming, Gamma distribution, Maintenance policy, Quality cost.

1 | Introduction

In production processes, operators or controllers examine the quality of products that may be a bad or a good form. Therefore, based on the identified state of the quality of products, different decisions should be made by decision-makers at the beginning of each production period. Such decisions include: i) doing nothing (continuing the production process), ii) accepting defective products, or (iii) renewing (replacing or repairing the machine). Also, the main objective of the production processes is to maximize the expected discounted value of profits. From a theoretical perspective, many studies, such as Monahan (1982), Ross (1983), White (1988), Valdez-Flores and Feldman (1989), Scarf (1997), and Wang (2002), have investigated production processes using a Markov decision-making model.

The optimal management of machine replacement is an essential problem in industries. Valdez-Florezand Feldman (1989) presented a review paper focusing on investigating failure distribution with constant parameters. Wilson and Popova (1998) provided a Bayesian parametric analysis considering parameters with random variables. Grosfeld-Nir (2007) investigated decision-making on machine

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replacement using a Markov model. Abdi and Taghipour (2019) developed an economic and environmental repair/ replacement model. Jafarian-Namin et al. (2021) proposed an integrated model including quality, maintenance, and production decisions considering delayed monitoring and ARMA control chart approaches. Hu et al. (2021) proposed a maintenance method considering a scheduled preventive replacement process. They examined the effect of an age-based preventive replacement policy on repairing a production process. Lio et al. (2021) proposed a data-driven Markov Decision Process to optimize repairing and replacing medical equipment decisions. Rebaiaia and Aït-Kadi (2021) analyzed three maintenance policies, including minimal renovations at defeat, replacement with total renewal at the first defeat, and replacement with complete renewal at all failure. Min (2021) investigated decision-making in modern sequential using dynamic programming. Pongha et al. (2022) examined the best production system considering a machine with time-varying failure and a single good. Pongha et al. (2022) analyzed the impacts of machine failures and repairs on the production rate of a hybrid system of producing and repairing. Recently, Yousefi et al. (2022) improved a new dynamic maintenance policy by applying a deep reinforcement learning approach. Wang (2022) improved an integrated queueing inventory and reliability model for controlling replacement and production processes considering time-dependent performance measures.

In this paper, we consider Bayesian inference, dynamic programming, and cost and probability functions to determine the optimal policy. The main objectives of this study are to: i) minimize costs and ii) maximize the probability of correct decision-making.

In addition, the contributions of this study are to: (i) design a decision system to find the optimal decision about machine maintenance by applying Bayesian inference and dynamic programming, (ii) make a decision among tube strategies, including replacing the machine, continuing the production process, and repairing the machine to attain the better performance of the production system and achieve a minimum hazard rate.

The rest of this research is as follows: the assumptions of this study are discussed in Section 2. In Section 3, the notations of the model are defined. Section 4 provides model formulations. In Section 5, a solution algorithm is proposed. A numerical example is provided in Section 6, and finally, Section 7 concludes the paper and discusses future research directions.

2. Assumptions

In this study, it is assumed that the random variable λ follows a Gamma distribution. Three policies are categorized as the optimal decisions in each period as follows: i) replacing the machine, ii) continuing the production process, and iii) repairing the machine. The best policy can be obtained by applying both the dynamic model and the Bayesian estimation method.

In this paper, we extend the machine replacement model proposed in the study of Niaki and Fallahnezhad (2007). They used dynamic programming and Bayesian inference approaches for decision-making in the production environment. In addition, they assumed that the rate of producing defective goods in each stage is fixed. Conversely, our study considers that the rate of producing faulty goods is not fixed. Specifically, if the "producing process" decision is made, then the rate of producing defective goods in the next step will increase from λ to $\phi.\lambda$. This is because the maintenance process is not done, and accordingly, the number of defective items will increase. Moreover, if the "repairing the machine" decision is made, then the rate of producing defective items will decrease form $\lambda$to $\varphi.\lambda$, because the maintenance work is done and therefore, the number of defective items will decrease. Note that $\phi > 1$ and $\varphi < 1$.

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It is assumed that the producing time of the defective products follows an exponential distribution with a hazard rate parameter λ. Note that t_i shows the producing time between the two successive defective products, and m denotes the number of defective items. The following posterior distribution is formulated based on the study by Nair et al. (2001).

$$f(\lambda) \in Gamma\left(\alpha = m, \beta = \sum_{i=1}^{m} t_i\right) \quad (1)$$

in which *f (.)* is the probability distribution of λ. Additionally, system costs include the (i) replacing the machine cost, (ii) continuing the production process cost, and (iii) repairing the machine cost.

3. Notations

The notations of this paper are defined as follows;

n: Remained stages in decision-making (stage variable in dynamic programming)

λ: The hazard rate (state variable in the dynamic programming)

t_i: The time between the production of $(i-1)^{th}$ and $(i)^{th}$ defective products in a production cycle

m: The number of defective products

f: The probability density function of λ

R: The machine replacement cost

T: The machine repair cost

C: The unit cost of defective goods in an order

$V_n(\lambda)$: The cost related to λ when there exist n stages in the decision-making

$W_n(\lambda)$: The correct choice probability related to λ when there exist n stages in the decision-making

d_n: The upper bound ofλ (If $\lambda \geq d_n$, then the production process will be stopped)

d_n': The lower threshold of λ (If $\lambda \leq d_n'$, then the production process will be continued)

δ_1: The accepted quality level (*AQL*) of the batch

δ_2: The lot tolerance proportion defective (*LTPD*) of the batch

λ_1: The AQL of the hazard rate

λ_2: The LTPD of the hazard rate

CS: The event of making the right decision

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ε_1: The probability of occurring type I error in decision-making

ε_2: The probability of occurring type II error in decision-making

H: The actual time of production

D: The size of a batch in an order

γ: The discount factor of stochastic dynamic programming

4. The model

The optimal decision should be selected from the three following decisions.

1) Replacing the machine,
2) Continuing the production process,
3) Repairing the machine and going to the next stage.

The probability of these policies is:

$P(\lambda \geq d_n)$: The probability of the machine replacing

$P(\lambda \leq d_n')$: The probability of continuing the production process

$1 - P(\lambda \geq d_n) - P(\lambda \leq d_n')$: The probability of repairing the machine

It is noted that $d_n \geq d_n'$because the third probability should not be negative.

$R.P(\lambda \geq d_n)$: The machine replacement cost

$C.H.\lambda.P(\lambda \leq d_n')$: The cost of continuing the production process

$\gamma.V_{n-1}(\lambda)$: The machine repair cost

Hence, the dynamic cost of the proposed model can be defined as Eq. (2).

$$E(Cost) = \begin{bmatrix} E(Cost|Replace\ the\ equipment)P(Replace\ the\ equipment) + \\ E\left(Cost\left|\begin{matrix} Continue\ a\ production \\ process\ by\ equipment \end{matrix}\right.\right) P\begin{pmatrix} Continue\ a\ production \\ process\ by\ equipment \end{pmatrix} + \\ E(Cost|Repair\ the\ equipment)P(Repair\ the\ equipment) \end{bmatrix} \quad (2)$$

The cost related to λ when there exist n stages in decision-making is formulated as follows:

$$V_n(\lambda) = \underset{d_n, d_n'}{Min} \begin{Bmatrix} \left(R + \gamma.V_{n-1}(\lambda_0)\right).P(\lambda \geq d_n) + \\ \left(C.H.\mu_\lambda + \gamma.V_{n-1}(\phi.\lambda)\right).P(\lambda \leq d_n') + \\ \left[1 - P(\lambda \geq d_n) - P(\lambda \leq d_n')\right].\left(\gamma.V_{n-1}(\varphi.\lambda) + T\right) \end{Bmatrix} \quad (3)$$

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It is assumed that if the production process is accepted, then the rate of producing defective items in the next stage will increase from λ to $\phi.\lambda$. If the repair machine decision is chosen, then the rate of making defective items will decrease form $\lambda$to $\varphi.\lambda$where $\phi > 1, \varphi < 1$.

To evaluate Equation (3), the probability distribution functions of random variables $\lambda' = \phi.\lambda$ and $\lambda'' = \varphi.\lambda$ should be determined as follows:

$$\left.\begin{array}{l} \lambda' = \varphi.\lambda \\ \lambda'' = \phi.\lambda \\ f(\lambda) \sim \Gamma\left(\alpha = m, \beta = \sum_{i=1}^{m} t_i\right) \end{array}\right\} \Rightarrow \begin{cases} f(\lambda') \sim \Gamma\left(\alpha, \dfrac{\beta}{\varphi}\right) & (4) \\ f(\lambda'') \sim \Gamma\left(\alpha, \dfrac{\beta}{\phi}\right) & (5) \end{cases}$$

Therefore, we have:

$$\begin{array}{l} V_{n-1}(\varphi.\lambda) = V_{n-1}(\lambda') \\ V_{n-1}(\phi.\lambda) = V_{n-1}(\lambda'') \end{array} \tag{6}$$

Also, it is assumed that $\lambda_0 \in \Gamma(1, \bar{\lambda}) \Rightarrow \lambda_0 \in Exp(\bar{\lambda})$ and $V_0(\lambda) = 100E(\lambda) = 100\mu_\lambda$. Regarding the CS definition, we have:

$$E(CS) = \begin{bmatrix} E(CS|Replace\ the\ equipment)P(Replace\ the\ equipment) + \\ E\left(CS \left| \begin{array}{l} Continue\ a\ production \\ process\ by\ equipment \end{array}\right.\right) P\left(\begin{array}{l} Continue\ a\ production \\ process\ by\ equipment \end{array}\right) + \\ E(CS|Repair\ the\ equipment)P(Repair\ the\ equipment) \end{bmatrix} \tag{7}$$

It is noted that $\frac{H\lambda_1}{D} = \delta_1$ and $\frac{H\lambda_2}{D} = \delta_2$. Hence, we have:

$$\begin{array}{l} P(CS|Replace\ the\ equipment) = \int_{\lambda_2}^{\infty} f(\lambda)\, d\lambda \\ P\left(CS \left| \begin{array}{l} Continue\ a\ production \\ process\ by\ equipment \end{array}\right.\right) = \int_0^{\lambda_1} f(\lambda)\, d\lambda \end{array} \tag{8}$$

To find the correct decision, the stochastic dynamic equation can be formulated as follows;

$$W_n(\lambda) = \underset{d_n, d_n'}{Max} \left\{ \begin{array}{l} P(\lambda \ge d_n).\left(\int_{\lambda_2}^{\infty} f(\lambda) d\lambda + \gamma.W_{n-1}(\lambda_0)\right) + \\ P(\lambda \le d_n').\left(\int_0^{\lambda_1} f(\lambda) d\lambda + \gamma.W_{n-1}(\phi\lambda)\right) + \\ \left[1 - P(\lambda \ge d_n) - P(\lambda \le d_n')\right].\left(\gamma.W_{n-1}(\varphi\lambda) + \int_{\lambda_1}^{\lambda_2} f(\lambda) d\lambda\right) \end{array} \right\} \tag{9}$$

Also, the logit function is used to obtain $W_0(\lambda)$as $W_0(\lambda) = \frac{1}{1+exp(\mu_\lambda)}$. In Equation (10), the best value of $H_n(\lambda)$ is obtained based on the thresholds of d_n and d_n' (Niaki and Fallahnezhad 2007).

$$H_n(\lambda) = \underset{d_n, d_n'}{Min} \left\{ \begin{array}{l} V_n(\lambda) \\ W_n(\lambda) \end{array} \right\} \tag{10}$$

To find the thresholds of d_n and d_n', the concept of first and second-type errors is used as follows.

1) If $\lambda \le \lambda_1 \rightarrow$ the stopping probability will be smaller than ε_1,

2) If $\lambda \ge \lambda_2 \rightarrow$ the continuing probability will be smaller than ε_2.

It is noted that $\frac{\alpha}{\beta}$ is the mean of Gamma distribution when a process is in a good state. Thus, we have:

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$$f(\lambda) \in \Gamma\left(\alpha = m, \beta = \sum_{i=1}^{m} t_i\right) \quad \Rightarrow \quad \frac{\alpha}{\beta} = \lambda_1 \quad \Rightarrow \quad \beta = \frac{\alpha}{\lambda_1} \tag{11}$$

In such a case, the probability of stopping the production process (Type I error) is as:

$$P(\lambda \geq d_n) = \int_{d_n}^{\infty} f(\lambda)\, d\lambda \leq \varepsilon_1 \quad \Rightarrow \quad 1 - F(d_n) \leq \varepsilon_1 \quad \Rightarrow \quad d_n \geq F^{-1}(1 - \varepsilon_1) \tag{12}$$

where $f(\lambda)$ is the probability function of Gamma distribution with parameters α and $\beta = \frac{\alpha}{\lambda_1}$. In addition, $F(d_n)$ is the cumulative probability distribution function of λ which is evaluated at d_n.

We define θ_1 as the threshold of d_n. Hence $F(d_n)$ is formulated as follows;

$$d_n \geq F^{-1}(1 - \varepsilon_1) = \theta_1 \tag{13}$$

Similarly, we define θ_2 as the threshold of $d_n{'}$. Thus, we have:

$$f(\lambda) \in \Gamma\left(\alpha = m, \beta = \sum_{i=1}^{m} t_i\right) \quad \Rightarrow \quad \frac{\alpha}{\beta} = \lambda_2 \quad \Rightarrow \quad \beta = \frac{\alpha}{\lambda_2} \tag{14}$$

In this case, the probability of continuing the production process (Type II error) is:

$$P(\lambda \leq d_n{'}) = \int_{0}^{d_n{'}} f(\lambda)\, d\lambda \leq \varepsilon_2 \quad \Rightarrow \quad F(d_n{'}) \leq \varepsilon_2 \quad \Rightarrow \quad d_n{'} \leq F^{-1}(\varepsilon_2) \tag{15}$$

where $f(\lambda)$ is the probability function of Gamma distribution with parameters α and $\beta = \frac{\alpha}{\lambda_2}$. Moreover, $F(d_n{'})$ is the cumulative probability distribution function of λ which is evaluated at $d_n{'}$.

We define θ_2 as the threshold of $d_n{'}$. Accordingly, $F(d_n)$ is formulated as follows;

$$d_n{'} \leq F^{-1}(\varepsilon_2) = \theta_2 \tag{16}$$

The possible cases regarding the optimal policy include different combinations of $d_n{'}$and $d_n$as:

$$\begin{cases} d_n = \infty, \\ d_n = \theta_1, \\ d_n{'} = 0, \\ d_n{'} = \theta_2 \end{cases}$$

Equation (17) expresses the mean of Gamma distribution with λ.

$$\frac{m}{\sum_{i=1}^{m} t_i} \tag{17}$$

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The optimal policy can be determined through Equation (18) because the best values of d_n and $d_n{}'$ meet the thresholds of optimal decision-making. In the following, decision rules are provided.

$$
\begin{aligned}
&1)\ \frac{\partial H_n(\lambda)}{\partial d_n} \le 0\ ,\ \ \frac{\partial H_n(\lambda)}{\partial d_n{}'} \le 0 \Rightarrow \begin{cases} d_n = \infty, \\ d_n{}' = \theta_2 \end{cases} \\
&\quad \Rightarrow \begin{cases} 1.1)\ if\ \dfrac{m}{\sum_{i=1}^{m} t_i} \le \theta_2 \Rightarrow \begin{pmatrix} Continue\ the\ production \\ process\ by\ equipment \end{pmatrix} \\ 1.2)\ Otherwise \Rightarrow \begin{pmatrix} Repair\ the\ equipment\ and \\ go\ to\ the\ next\ stage \end{pmatrix} \end{cases} \\
&2)\ \frac{\partial H_n(\lambda)}{\partial d_n} \le 0\ ,\ \ \frac{\partial H_n(\lambda)}{\partial d_n{}'} \ge 0 \Rightarrow \begin{cases} d_n = \infty, \\ d_n{}' = 0 \end{cases} \Rightarrow \begin{pmatrix} Repair\ the\ equipment\ and \\ go\ to\ the\ next\ stage \end{pmatrix} \\
&3)\ \frac{\partial H_n(\lambda)}{\partial d_n} \ge 0\ ,\ \ \frac{\partial H_n(\lambda)}{\partial d_n{}'} \le 0 \Rightarrow \begin{cases} d_n = \theta_1, \\ d_n{}' = \theta_2 \end{cases} \\
&\quad \Rightarrow \begin{cases} 3.1)\ if\ \theta_1 \le \dfrac{m}{\sum_{i=1}^{m} t_i} \le \theta_2 \Rightarrow \begin{pmatrix} Since\ d_n \ge d_n{}'\ this\ case \\ is\ not\ feasible\ and\ should \\ not\ be\ considered \end{pmatrix} \\ 3.2)\ if\ \theta_2 \le \dfrac{m}{\sum_{i=1}^{m} t_i} \le \theta_1 \Rightarrow \begin{pmatrix} Repair\ the\ equipment\ and \\ go\ to\ the\ next\ stage \end{pmatrix} \\ 3.3)\ if\ \theta_1, \theta_2 \le \dfrac{m}{\sum_{i=1}^{m} t_i} \Rightarrow (Replace\ the\ equipment) \\ 3.4)\ if\ \dfrac{m}{\sum_{i=1}^{m} t_i} \le \theta_1, \theta_2 \Rightarrow \begin{pmatrix} Continue\ the\ production \\ process\ by\ equipment \end{pmatrix} \end{cases} \\
&4)\ \frac{\partial H_n(\lambda)}{\partial d_n} \ge 0\ ,\ \ \frac{\partial H_n(\lambda)}{\partial d_n{}'} \ge 0 \Rightarrow \begin{cases} d_n = \theta_1, \\ d_n{}' = 0 \end{cases} \\
&\quad \Rightarrow \begin{cases} 4.1)\ if\ \theta_1 \le \dfrac{m}{\sum_{i=1}^{m} t_i} \Rightarrow (Replace\ the\ equipment) \\ 4.2)\ Otherwise \Rightarrow \begin{pmatrix} Repair\ the\ equipment\ and \\ go\ to\ the\ next\ stage \end{pmatrix} \end{cases}
\end{aligned} \tag{18}
$$

To solve the investigated model, we propose an algorithm in the next section.

5. The solution algorithm

To solve the considered problem, the following algorithm is proposed.

a) In the first stage (*n=1*), we define $H_n(\lambda)$ using Equation (10) and considering $\beta = \sum t_i$, $\alpha = m$, f, R, T, C, δ_1, δ_2, λ_1, λ_2, ε_1, ε_2, H, D and γ .

b) Using equations (13) and (16) and numerical integrations, the thresholds of d_1 and d_1' are determined as θ_1 and θ_2.

c) The best value of $H_n(\lambda)$ is determined regarding four cases $(d_1 = 0, d_1' = \infty)$,$(d_1 = 0, d_1' = \theta_2)$, $(d_1 = \theta_1, d_1' = \infty)$, and $(d_1 = \theta_1, d_1' = \theta_2)$.

d) We decide according to the value of $E(\lambda)$ (Equation (17)) and the thresholds.

e) Set $n = n + 1$ and determine the optimal value of $H_{n-1}(\lambda)$. Then, go to Step (a).

To evaluate the optimal value of $H_{n-1}(\lambda)$ in Step (e), we need to calculate the optimal values of $H_{n-2}(\lambda), H_{n-3}(\lambda), \dots,$ and $H_1(\lambda)$.

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In the following section, a numerical example is carried out to evaluate the effectiveness of the proposed model.

6. The numerical example

In this example, two decision-making stages are investigated. We assume that the number of defective products is $\alpha = m = 2$, and the summation time between the productions of two defective products in a row is $\beta = \sum_{i=1}^{m} t_i = 12$. The data of other parameters are:$R = 100$ (\$), $T = 50$ (\$), $C = 1$ (\$), $\delta_1 = 0.04$, $\delta_2 = 0.1$. $\lambda_1 = 0.16$, $\lambda_2 = 0.4$, $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.1$, $H = 1000$, $D = 4000$, $\gamma = 0.8$. Moreover, the values of $\bar{\lambda}, \varphi, \phi$ are 1000, 0.5 and 1.1, respectively.

Considering Equation (1), it is obtained that $f(\lambda) \approx Gamma(\alpha = m = 2, \beta = \sum t_i = 12)$, and considering the first step of the algorithm$(n = 1)$, Equation (10) is formulated as:

$$H_1(\lambda) = \frac{\begin{pmatrix}\left(R + \gamma . V_0(\lambda_0)\right). P(\lambda \geq d_1) + \left(C.H.\mu_\lambda + \gamma . V_0(\phi . \lambda)\right). P(\lambda \leq d_1') + \\ [1 - P(\lambda \geq d_1) - P(\lambda \leq d_1')].(\gamma . V_0(\varphi . \lambda) + T)\end{pmatrix}}{\begin{pmatrix}P(\lambda \geq d_1).\left(\int_{\lambda_2}^{\infty} f(\lambda)\,d\lambda + \gamma . W_0(\lambda_0)\right) + P(\lambda \leq d_1').\left(\int_0^{\lambda_1} f(\lambda)\,d\lambda + \gamma . W_0(\phi\lambda)\right) + \\ [1 - P(\lambda \geq d_1) - P(\lambda \leq d_1')].\left(\gamma . W_0(\varphi\lambda) + \int_{\lambda_1}^{\lambda_2} f(\lambda) d\lambda\right)\end{pmatrix}} \tag{19}$$

The terms in Equation (19) are obtained as follows;

$$V_0(\lambda_0) = 0.1 \qquad \mu_\lambda = \frac{\alpha}{\beta} = 0.1667 \qquad V_0(\phi . \lambda) = 18.3333$$

$$V_0(\varphi . \lambda) = 8.3333 \qquad \int_{\lambda_2}^{\infty} f(\lambda)\,d\lambda = 0.9994 \qquad \int_0^{\lambda_1} f(\lambda)\,d\lambda = 8.8103E - 05$$

$$W_0(\phi\lambda) = 0.4542 \quad W_0(\varphi\lambda) = 0.4791 \quad W_0(\lambda_0) = 0.4997 \quad \int_{\lambda_1}^{\lambda_2} f(\lambda) d\lambda = 0.00045526$$

We use $\lambda_0 \in Exp(\bar{\lambda})$, $V_0(\lambda) = 100\mu_\lambda$, and $W_0(\lambda) = \frac{1}{1+exp(\mu_\lambda)}$ to obtain the values of parameters mentioned above. Therefore, we have:

$$H_1(\lambda) = \frac{\begin{pmatrix}(100 + 0.8 * 0.1). P(\lambda \geq d_1) + (1 * 1000 * 0.1667 + 0.8 * 18.3333). P(\lambda \leq d_1') + \\ [1 - P(\lambda \geq d_1) - P(\lambda \leq d_1')].(0.8 * 8.3333 + 50)\end{pmatrix}}{\begin{pmatrix}(0.9994 + 0.8 * 0.4997). P(\lambda \geq d_1) + (8.81103E - 5 + 0.8 * 0.4542). P(\lambda \leq d_1') + \\ [1 - P(\lambda \geq d_1) - P(\lambda \leq d_1')].(0.8 * 0.4791 + 0.000445526)\end{pmatrix}}$$

In the second step, using equations (13) and (16), we have:

$$f(\lambda) \in \Gamma\left(\alpha = m = 2, \beta = \sum_{i=1}^{m} t_i = 12\right) \Rightarrow \frac{\alpha}{\beta} = \lambda_1 \Rightarrow \beta = \frac{\alpha}{\lambda_1} \qquad d_n' \geq F^{-1}(1 - \varepsilon_1) = \theta_1$$

$$f(\lambda) \in \Gamma\left(\alpha = m = 2, \beta = \sum_{i=1}^{m} t_i = 12\right) \Rightarrow \frac{\alpha}{\beta} = \lambda_2 \Rightarrow \beta = \frac{\alpha}{\lambda_2} \qquad d_n' \leq F^{-1}(\varepsilon_2) = \theta_2$$

which are evaluated by Excel 2019 for $\theta_1 = 60$ and $\theta_2 = 3$.

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In in Step 3, the objective function is evaluated under different thresholds of d_1, d_1', and then the best values of d_1 and d_1', which minimize the objective function, are selected as follows;

$$\left.\begin{matrix} d_1 = \infty \\ d_1' = 0 \end{matrix}\right\} \rightarrow H_1(0.1667) = 147.647 \qquad \left.\begin{matrix} d_1 = \infty \\ d_1' = 3 \end{matrix}\right\} \rightarrow H_1(0.1667) = 156.4736$$

$$\left.\begin{matrix} d_1 = 60 \\ d_1' = 0 \end{matrix}\right\} \rightarrow H_1(0.1667) = 137.5112 \qquad \left.\begin{matrix} d_1 = 60 \\ d_1' = 3 \end{matrix}\right\} \rightarrow H_1(0.1667) = 145.471$$

Hence, the minimum value of H_1 is obtained as 137.5112 and the best values of d_1 and d_1' are 60 and 0, respectively.

In the fourth step, based on the obtained values of the expected mean of time between productions of defective products (i.e.,0.1667) and $d_1 = 60 \geq \frac{m}{\sum t_i} = 0.1667 \geq d_1' = 0$, we should repair the machine and then go to the next stage.

In Stage 2 ($n = 2$), we assume $\alpha = m = 5$ and $\beta = \sum t_i = 110$. According to Equation (1), it is obtained that $f(\lambda) \approx Gamma(\alpha = m = 5, \beta = \sum t_i = 110)$, and Equation (10) is formulated as follows.

$$H_2(\lambda) = \frac{\left(\begin{matrix} \left(R + \gamma.V_1(\lambda_0)\right).P(\lambda \geq d_2) + \left(C.H.\mu_\lambda + \gamma.V_1(\phi.\lambda)\right).P(\lambda \leq d_2') + \\ [1 - P(\lambda \geq d_2) - P(\lambda \leq d_2')].(\gamma.V_1(\varphi.\lambda) + T) \end{matrix}\right)}{\left(\begin{matrix} P(\lambda \geq d_2).\left(\int_{\lambda_2}^{\infty} f(\lambda)\,d\lambda + \gamma.W_1(\lambda_0)\right) + P(\lambda \leq d_2').\left(\int_0^{\lambda_1} f(\lambda)\,d\lambda + \gamma.W_1(\phi\lambda)\right) + \\ [1 - P(\lambda \geq d_2) - P(\lambda \leq d_2')].\left(\gamma.W_1(\varphi\lambda) + \int_{\lambda_1}^{\lambda_2} f(\lambda)d\lambda\right) \end{matrix}\right)}$$

In the second step, considering equations (13) and (16), we have:

$$f(\lambda) \in \Gamma\left(\alpha = m = 5, \beta = \sum_{i=1}^{m} t_i = 110\right) \Rightarrow \frac{\alpha}{\beta} = \lambda_1 \Rightarrow \beta = \frac{\alpha}{\lambda_1} \qquad d_n' \geq F^{-1}(1 - \varepsilon_1) = \theta_1$$

$$f(\lambda) \in \Gamma\left(\alpha = m = 5, \beta = \sum_{i=1}^{m} t_i = 110\right) \Rightarrow \frac{\alpha}{\beta} = \lambda_2 \Rightarrow \beta = \frac{\alpha}{\lambda_2} \qquad d_n' \leq F^{-1}(\varepsilon_2) = \theta_2$$

which are evaluated by Excel 2019 for $\theta_1 = 287$ and $\theta_2 = 31$.

Then in Step 3, the objective function is evaluated under different possible thresholds of d_1, d_1', and then the best values of d_1 and d_1', which minimize the objective function, are chosen as:

$$\left.\begin{matrix} d_2 = \infty \\ d_2' = 0 \end{matrix}\right\} \rightarrow H_2(0.045455) = 285.1428 \qquad \left.\begin{matrix} d_2 = \infty \\ d_2' = 31 \end{matrix}\right\} \rightarrow H_2(0.045455) = 281.1427272$$

$$\left.\begin{matrix} d_2 = 278 \\ d_2' = 0 \end{matrix}\right\} \rightarrow H_2(0.045455) = 121.99234 \qquad \left.\begin{matrix} d_2 = 278 \\ d_2' = 31 \end{matrix}\right\} \rightarrow H_2(0.045455) = 111.9923086$$

Hence, the minimum value of H_2 is 111.9923086 and the optimal values of d_1 and d_1' are 278 and 31, respectively.

In Step 4, based on the value of the mean of time between productions of defective products (i.e., 0.045455), and $d_1 = 278, d_1' = 31 \geq \frac{m}{\sum t_i} = 0.045455$, we should continue the production process. It is noteworthy that carrying out the maintenance work is not necessary.

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7. Conclusions

In this research, we proposed the finite horizon single-item maintenance optimization structured for preventive maintenance and quality control. One Bayesian model was developed to model the distribution of time between the productions of two defective goods in a row. We also formulated the system as a dynamic programming model, and derived some properties of the best function, which enabled us to efficiently search the optimal maintenance policy, minimize the expected total discounted system cost, and maximize the total discounted probability of correct decisions simultaneously. This study can be extended in several paths. First, this research can be developed in an uncertain environment. Second, for future research, it will be interesting to consider the other probability distributions, such as Bath tub curve probability distribution function, and compare the effects of different probability distribution functions.

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