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A Novel Multi-Stage Stochastic Portfolio Optimization Model Under Transaction Costs

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Abstract

In this study, a multi-stage stochastic portfolio selection procedure is proposed. Specifically, to deal with market uncertainty, a three-pronged goal and a Wealth-Mean absolute semi deviation-Liquidity (WML) portfolio selection model based on a scenario tree are developed. Due to duration, continuity of the horizon, and uncertainty, the scenario tree is an ideal tool for modeling multi-period portfolio problems. Wealth, risk, asset investment threshold, transaction cost, and liquidity are important variables for the problem under investigation. This study uses rebalancing and mean absolute semi-deviation as measures of portfolio risk. A Node-Based Modelling (NBM) method is used to develop effective investment strategies for the long-term investment horizon. In addition, using a goal programming approach, the investigated multi-objective model converts to a single-objective model. To demonstrate the effectiveness of the proposed model, a real-world empirical application with data set from a Tehran stock market (TSE) is carried out. The experimental findings indicate the applicability of the developed model.

Keywords: Node based modelling, Multi-period investment, Portfolio selection, Scenario tree.

1 | Introduction

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Investment in financial markets can achieve two goals: (i) maximizing the anticipated utility of return and (ii) limiting risk exposure of return. An investment portfolio is one of the methods for achieving these objectives. Markowitz (1952) introduced a mean-variance model for portfolio selection, which has two primary goals: maximize anticipated return and reduce expected risk. This model is the foundation for the current portfolio theory. In practical situations, portfolio strategies are often multi-period, allowing investors to reassess their investment strategy. However, several expansions have been offered in a single-period horizon (Peykani et al., 2022). Mulvey and Vladimirou (1989) examined a multi-period stochastic programming model for the portfolio selection problem. Mulvey and Ziemba (1995) and Ziemba (1998) concentrated their study on the issue of scenario tree consumption and investment. Mulvey et al. (1997) introduced a nonlinear model that extracts asset/liability management for efficiently controlling risk over long periods in recent research. Dupacova (1999) proposed approaches for analyzing results obtained by solving stochastic algorithms based on asymptotic and robust statistics, the moment problem, and parametric optimization findings. Mulvey

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and Shetty (2004) presented a framework for modeling financial planning problems using multi-stage optimization and interior-point approaches with the scenario to deal with uncertainty.

Pinar (2007) developed multi-stage portfolio selection models with a linear composite objective and a simulated market model to deal with market unpredictability. Gulpinar and Rustem (2007) introduced a multi-period mean-variance optimization framework for the worst-case design of the scenario tree's stochastic characteristics. Edirisinghe and Patterson (2007) investigated a stochastic multi-stage programming model using block separable recourse structure in layered L-shaped decomposition. Sakar and Koksalan (2013) studied a stochastic programming technique for solving the multi-criteria multi-period portfolio optimization issue. They used a single index model (SIM) to estimate stock returns from a market-representative index and a random walk model to construct scenarios for the index return potential values. Najafi and Mushakhian (2015) investigated a multi-stage stochastic mean-semivariance-CVaR model considering scenario trees to tackle uncertainty. Additionally, they created a hybrid metaheuristic that uses both a genetic algorithm (GA) and a particle swarm optimization method (PSO) to find a solution. Chen et al. (2016) explored a deterministic convex programming model for a multi-period portfolio selection problem with terminal distortion risk measure using the scenario tree approach. To handle uncertainty, Mohebbi and Najafi (2018) developed a bi-objective mean-VaR portfolio selection model by fusing the fuzzy credibility theory with scenario trees. Nouri and Mohammadi (2018) suggested a randomized method for transforming uncertainty into a state of certainty and combined it with agreeing to prepare for a particular goal. To deal with market uncertainty, Liu and Chen (2018) suggested two multi-period robust risk metrics using a regime-switching framework and scenario trees. Nasaz et al. (2020) investigated a multi-period optimization problem using a meta-heuristic method (i.e., Non-dominated Sorting Genetic Algorithm II (NSGA II)). To demonstrate the effectiveness of the suggested technique, they also employed several quantitative performance metrics.

Prior studies have indicated that stochastic programming models are flexible tools to describe financial optimization problems under uncertainty. In the literature, various formulations have been used for the multi-stage financial problem (Kall, Wallace, and Kall, 1994). Using multi-stage stochastic programming, Carino et al. (1994) investigated an asset/liability management issue. For the best distribution of assets, a hybrid simulation/tree multi-period stochastic programming model has been developed in the study of Hibiki (2006). In multi-period stochastic programming models, random parameters are often modeled using scenarios. Based on a tree structure, scenarios are built (Mulvey and Ziemba, 1995). The conditional character of the scenario tree is taken into account, which is based on the extension of the decision space. At each node, conditional judgments are made within the restrictions of the modeling. The number of choice factors and conditions in the scenario tree may increase exponentially to guarantee that the generated representative set of scenarios adequately covers the set of possibilities.

To sum up, scholars have employed variance as a risk indicator. Due to asymmetric return distributions, the demand of investors, and the measurement of actual investment risk, semi-deviation-based metrics of downside risk should be substituted by variance. Risk management options use semi-deviation to describe the loss over a particular period with a given level of confidence (Ji and Lejeune, 2018). Although several studies have investigated the multi-stage stochastic portfolio selection issue, most of them have employed one objective function. Specifically, they have primarily considered the mean or risk of the portfolio as the objective function, and they have ignored considering stock liquidity as the objective function. Additionally, the majority of these studies were nonlinear, and they have not determined a global solution. Thus, investors cannot trust the chosen portfolio.

In this paper, our proposed model includes three objectives. The first objective is considered wealth, the second objective is considered semi-deviation (as a downside risk measure), and the third objective is the liquidity of the portfolio. Also, the transaction cost is taken into account. Our proposed linear model has the global solution. Our study adds a new perspective on multi-stage stochastic problems. Moreover, a goal programming (GP) technique is employed to obtain the optimal strategy. Specifically,

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a scenario tree approach and a GP method are used to transform the proposed stochastic multi-objective model into a crisp single-objective problem.

The remainder of this paper is structured as follows. In Section 2, a multi-period portfolio optimization model is developed considering transaction costs and limitations. Section 3 explains a goal programming approach to solve the suggested model. Section 4 provides a numerical example, and discusses findings and managerial insights. Section 5 concludes the paper and provides future research paths.

2. The multi-stage portfolio optimization model

2.1. Problem statement

In this study, a multi-stage portfolio optimization model is considered under a stochastic environment via I risky assets. A planning horizon consists of T stages. A transaction takes place at discrete time points. Time intervals can vary from minutes to years, and decisions are made at the beginning of the stages. The current date is deemed as the first stage. Revenue from the sales is added to the budget, and expenses from the purchases are detracted from the budget. At time t+1, based on the realized returns over (t, t+1] the investor's holdings are updated. At the end of period T, an investor collects his final wealth W_T. The investor's goal is to manage a portfolio of assets to maximize the expected utility of final wealth $E[U(W_T)]$. Uncertainty is modeled through scenarios, and each scenario describes a possible realization of all uncertain parameters in the model. Each scenario S at period t has a probability p_t^s, where $p_t^s > 0$ and $\sum_{s=1}^{S} p_t^s = 1$.

In a dynamic model, a suitable way to represent uncertainty is a scenario tree because it creates information visible on the real value of the uncertain parameters under stages. A scenario is a route from the root to a leaf. Any node of the tree, corresponding to time t, stands for a possible state of the world, and it is crystal clear that all scenarios, passing these nodes, have the same history in periods $0,1,2,\ldots,T-1$.

2.2. Sets, parameters, and decision variables

Sets

i	risky asset $i = 1,2,3,\ldots,I$;
t	investment period $t = 1,2,3,\ldots,T$;
s	scenario $s = 1,2,3,\ldots,s_t$;
φ	node number in each scenario;
φ'	later node of node φ;
ϕ_t	the number of nodes at the beginning of period t;

Parameters

r_{it}^{φ}	return rate of risky asset i at period t and node φ;
r_{pt}^{φ}	return rate of portfolio at period t and node φ;
ub_{it}^{φ}	upper bound of x_{it}^{φ};
lb_{it}^{φ}	lower bound of x_{it}^{φ};
l_{it}^{φ}	the liquidity of risky asset i at period t and node φ;
S_t	the number of scenarios at period t that branch from each node;
p_t^s	the occurrence probability of scenario s at period t; ($\sum_{s=1}^{S_t} p_t^s = 1$)
c_{it}^{φ}	unit transaction cost of risky asset i at period t and node φ;

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Variables

Liq_t^{φ}	liquidity of portfolio at period *t* and node φ;
SD_t	semi deviation as risk at period t;
x_{it}^{φ}	invest of risky asset i at period t for node φ, as a decision variable;
X_t^{φ}	portfolio at period t for node φ, i.e.,$X_t^{\varphi} = (x_{1t}^{\varphi}, x_{2t}^{\varphi}, \dots x_{It}^{\varphi})$;
W_t^{φ}	available net wealth of Scenario s at the beginning of period t;

2.3. Mathematical definition of the model

The suggested model in this work has three goals: the first goal is wealth, the second goal is to evaluate downside risk using semi-deviation, and the third goal is to quantify portfolio liquidity. The suggested linear model offers a comprehensive solution. Additionally, the transaction cost is considered, leading to the proposal of a novel viewpoint on multi-stage stochastic situations. The goal programming (GP) approach is then used to find the best action. The suggested stochastic multi-objective model is reduced to a clear single-objective issue using GP and a scenario tree technique.

2.3.1. Objective functions

- Maximizing Terminal Wealth

In the multi-period portfolio optimization problem, the wealth without cost transaction at period *t* for nodeφ can be calculated by $W_t^{\varphi} = \sum_{i=1}^{I} r_{it}^{\varphi} x_{it}^{\varphi}$. In 1982, Patel and Subrahmanyam (1982) proved that ignoring the transaction cost during portfolio trading often leads to an inefficient portfolio. The transaction cost is defined as a V-shape function of differences between tth and $t-1$th period portfolio (Markowitz and Todd, 2000), the transaction cost of the risky asset at period t for node φ is $c_{it}^{\varphi}\left|x_{it}^{\varphi} - x_{it-1}^{\varphi'}\right|$ with $\varphi \epsilon \phi_t$ and $\acute{\varphi} \epsilon \phi_{t-1}$. The total transaction cost of portfolio at period *t* under the scenario is expressed as $C_t^{\varphi} = \sum_{i=1}^{I} c_{it}^{\varphi}\left|x_{it}^{\varphi} - x_{it-1}^{\varphi'}\right|$.

Hence, the net wealth of the portfolio at period *t* and node φ can be denoted as

$$W_t^{\varphi} = \sum_{i=1}^{I} r_{it}^{\varphi} x_{it}^{\varphi} - \sum_{i=1}^{I} c_{it}^{\varphi}\left|x_{it}^{\varphi} - x_{it-1}^{\varphi'}\right| \qquad \varphi \epsilon \phi_t, \varphi' \epsilon \phi_{t-1} \text{ and } t \in T \tag{1}$$

The terminal wealth as the objective is expressed:

$$E(W_T^{\varphi}) = E\left(\sum_{i=1}^{I} r_{it}^{\varphi} x_{iT}^{\varphi} - \sum_{i=1}^{I} c_{iT}^{\varphi}\left|x_{iT}^{\varphi} - x_{iT-1}^{\varphi'}\right|\right) \qquad \varphi \epsilon \phi_T, \varphi' \epsilon \phi_{T-1} \tag{2}$$

- Minimizing Risk

The risk of a portfolio may be measured using different approaches. Measures of downside risk disregard any upward departures from anticipated return (as they are to the investor's advantage) and are solely concerned with a portfolio's losses. In the literature, there are several metrics of downside risk, including mean absolute semi-deviation (MASD), value-at-risk (VaR), and conditional value-at-risk (CVaR) (Mehlawat et al., 2021).

The expected value of the negative deviations from the anticipated return on an asset is the mean absolute semi-deviation. Since most investors view risk as the potential for an asset to perform below its intended rate of return, MASD fits with this understanding of risk. As suggested by Markowitz (1959), variance is not used but instead minimizes the MASD of the portfolio, which penalizes the downside

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risk by $E\left[\left[\sum_{i\in I}\sum_{j\in J}(\mu_j - R_j)x_{ij}\right]^+\right]$ that i and j show scenarios and assets, respectively. Also,$\mu_j$and $R_j$show expected return and asset return, respectively (Ji & Lejeune, 2018). Now, the relation is customized for period t:

$$SD_t = E\left[\left[\sum_{\varphi\in\phi_t}\sum_{i\in I}(\mu_t^{\varphi} - r_{it}^{\varphi})x_{it}^{\varphi}\right]^+\right] \qquad t\in T \qquad (3)$$

That $\mu_t^{\varphi} = \sum_{i=1}^{I} E\left(r_{it}^{\varphi}\right)$ for $\varphi\epsilon\phi_t$and $t \in T$.

To consider various scenarios in the multi-period portfolio optimization problem, the semi-deviation function can be utilized to minimize the risk of the investigated portfolio as follows:

$$\sum_{t\in T} SD_t = \sum_{t\in T} E\left[\left[\sum_{\varphi=1}^{\phi_t}\sum_{i=1}^{I}(\mu_t^{\varphi} - r_{it}^{\varphi})x_{it}^{\varphi}\right]^+\right] \qquad (4)$$

- Liquidity

In decision-making on portfolio investment, one of the key elements that should be considered is liquidity for investors. It measures the degree of probability that investors will convert an asset into income. Investors prefer assets with higher liquidity because their returns tend to rise over time. Generally, liquidity is measured by the turnover rate of assets. According to Fong et al. (2017), the closing percent quoted spread from Chung and Zhang (2014) is the best spread proxy for capturing changes in effective and quoted spreads. The closing percent quoted spread (*Spread*) of stock *i* on day *t* is defined as

$$Spread_{it} = \frac{Ask_{it} - Bid_{it}}{M_{it}} \qquad (5)$$

where $Ask_{it}$is the closing ask price of risky asset *i* at period *t*, Bid_{it} is the closing bid price of *i* at period *t*, and $M_{it}$is the mean of $Ask_{it}$and Bid_{it} (Ma, Anderson, & Marshall, 2018). Now for using the spread in this modeling, it should be extended to cover nodes in the scenario tree. Therefore, $Spread_{it}$is changed to $Spread_{it}^{\varphi}$, and its relation is changed to the following equation:

$$Spread_{it}^{\varphi} = \frac{Ask_{it}^{\varphi} - Bid_{it}^{\varphi}}{M_{it}^{\varphi}} \qquad (6)$$

The portfolio liquidity for each node is expressed as:

$$Liq_t^{\varphi} = E\left(\sum_{i=1}^{I} l_{it}^{\varphi}x_{it}^{\varphi}\right) \qquad \varphi\epsilon\phi_t, t\in T \qquad (7)$$

For calculating overall liquidity as the objective, the following relation is defined:

$$Max \sum_{t=1}^{T}\sum_{\varphi=1}^{\phi_t} Liq_t^{\varphi} = Max \sum_{t=1}^{T}\sum_{\varphi=1}^{\phi_t} E\left(\sum_{i=1}^{I} l_{it}^{\varphi}x_{it}^{\varphi}\right) \qquad (8)$$

2.3.2. Constraints

- Budget

At the beginning of the first period, the budget constraint is:

$$\sum_{i=1}^{I} x_{i1}^{\varphi} + \sum_{i=1}^{I} c_{i1}^{\varphi}x_{i1}^{\varphi} = W \qquad \varphi\epsilon\phi_1 \qquad (9)$$

- Cash Flows

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Cash flow Constraints are between periods, which is as relationships between each node, and later nodes are determined by

$$\sum_{i=1}^{I} x_{it}^{\varphi} = \sum_{i=1}^{I} E(r_{it}^{\varphi})\, x_{it}^{\varphi'} - \sum_{i=1}^{I} c_{it}^{\varphi} \left| x_{it}^{\varphi} - x_{it-1}^{\varphi'} \right| \qquad \varphi \epsilon \phi_t, \varphi' \epsilon \phi_{t-1} \text{and } t \in T \qquad (10)$$

- Minimum Return

The minimum return at each node, which the portfolio return must achieve, is expressed as

$$\sum_{i=1}^{I} r_{it}^{\varphi} x_{it}^{\varphi} \geq \mu \qquad t \in T \qquad (11)$$

- Threshold

The minimum and maximum amount of investment in the i-th asset at period t and node can be determined as

$$lb_{it}^{\varphi} \leq x_{it}^{\varphi} \leq ub_{it}^{\varphi} \qquad i \in I, \varphi \epsilon \phi_t, t \in T \qquad (12)$$

- No short selling

No short selling of assets at each period t is expressed as

$$x_{it}^{\varphi} \geq 0 \qquad i \in I, \varphi \epsilon \phi_t, t \in T \qquad (13)$$

- The proposed model

Over the entire investment horizon, the investor intends to obtain the most significant final wealth and minimizes the risk simultaneously. Thus, the multi-period portfolio optimization model is as follows:

$$\begin{array}{ll}
Max & E(\sum_{i=1}^{I} r_{iT}^{\varphi} x_{iT}^{\varphi} - \sum_{i=1}^{I} c_{iT}^{\varphi} \left| x_{iT}^{\varphi} - x_{iT-1}^{\varphi'} \right|) \\
Min & \sum_{t=1}^{T} E\left[\left[\sum_{\varphi=1}^{\phi_t} \sum_{i=1}^{I} (\mu_t^{\varphi} - r_{it}^{\varphi}) x_{it}^{\varphi}\right]^{+}\right] \\
Max & \sum_{t=1}^{T} \sum_{\varphi=1}^{\phi_t} E(\sum_{i=1}^{I} l_{it}^{\varphi} x_{it}^{\varphi})
\end{array} \qquad (14)$$

s.t.

$$\begin{array}{ll}
\sum_{i=1}^{I} x_{i1}^{\varphi} + \sum_{i=1}^{I} c_{i1}^{\varphi} x_{i1}^{\varphi} = W & \varphi \epsilon \phi_1 \\
\sum_{i=1}^{I} x_{it}^{\varphi} = \sum_{i=1}^{I} E(r_{it}^{\varphi})\, x_{it}^{\varphi'} - \sum_{i=1}^{I} c_{it}^{\varphi} \left| x_{it}^{\varphi} - x_{it-1}^{\varphi'} \right| & \varphi \epsilon \phi_t, \varphi' \epsilon \phi_{t-1} \text{and } t \in T \\
\sum_{i=1}^{I} r_{it}^{\varphi} x_{it}^{\varphi} \geq \mu & t \in T \\
lb_{it}^{\varphi} \leq x_{it}^{\varphi} \leq ub_{it}^{\varphi} & \varphi \epsilon \phi_t, t \in T, i \in I \\
x_{it}^{\varphi} \geq 0 & i \in I, \varphi \epsilon \phi_t, t \in T
\end{array}$$

3. The goal programming technique

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For multiple-objective decision-making (MODM) issues where specific objectives are incompatible and non-commensurable, there are some strategies and algorithms available. One of the most valued, powerful, and practical techniques for solving MODM issues in goal programming (GP). Regarding the underlying distance metric, lexicographic, weighted, and Chebyshev goal programming are the three main variations of GP. Additionally, GP may be divided into fuzzy, integer, binary, and fractional goal programming depending on the mathematical structure of the decision variables and, or objectives (Charnes & Cooper, 1977).

Numerous GP versions developed to deal with the uncertainty surrounding securities for portfolio selection, are typically based on fuzzy theories and probability. Additionally, several scholars indicated that GP is a helpful method for solving multiple-objective problems. Peykani et al. (2021) introduced a unique fuzzy multi-period multi-objective portfolio optimization model using GP to handle two problems. First, it can be employed even in situations with ambiguous data and practical limitations. Second, in the fuzzy uncertainty setting, it was a multi-period portfolio management approach. Two intuitionistic fuzzy portfolio selection models for optimistic and pessimistic situations were described by Gupta et al. in 2019. For a portfolio selection issue, Yu et al. (2021) created a brand-new, unified intuitionistic fuzzy multi-objective linear programming model. Therefore, GP is used as the most feasible strategy.

In the following, the multiple objective linear programming (MOLP) problem is expressed in which c, a, and b, are the objective function coefficient, the technological coefficient, and the right-hand-side, respectively.

$$\begin{aligned} &Max \ \sum_{j=1}^{J} c_{j1}x_j \\ &\quad \vdots \\ &Max \ \sum_{j=1}^{J} c_{j1}x_j \\ &S.t. \ \sum_{j=1}^{J} a_{ij}x_j \le b_i \quad ,\forall i \\ &\qquad x_j \ge 0, \quad \forall j \end{aligned} \tag{15}$$

Assume that a set of N goals $\{\Psi_1, \dots, \Psi_n, \dots, \Psi_N\}$ is specified by the decision maker (DM) for objective functions. Goal programming tries to achieve an optimal solution "as close as possible" to the set of specified goals that may not be simultaneously attainable. The equivalent weighted GP mathematical formulation to the above MOLP is written as follows:

$$\begin{aligned} &Min \ \sum_{n=1}^{N} (\alpha_n^- \xi_n^-, \alpha_n^+ \xi_n^+) \\ &S.t. \\ &\sum_{j=1}^{J} c_{jk}x_j + \xi_n^- - \xi_n^+ = \Psi_n, \quad \forall n \\ &\sum_{j=1}^{J} a_{ij}x_j \le b_i, \quad \forall i \\ &x_j, \xi_n^-, \xi_n^+ \ge 0, \quad \forall j, n \end{aligned} \tag{16}$$

It should be elucidated that non-negative variables ξ_n^- and ξ_n^+ are deviational variables of goal k. Also, α_n^- and α_n^+ are weights assigned to the deviational variables of goal k are determined by the decision maker. The weighted GP mathematical formulation can be expanded to handle the objectives (goals) at different priority classes and levels.

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4. Implementation and results

In this part, a real-world case study from the Tehran stock exchange is investigated. A multi-period multi-objective portfolio optimization model (TSE) is used to extract the data set of 20 stocks over three periods. Additionally, each node generates two possibilities once per period. The first node is produced at the start of the first period. Two possibilities are then made. There are four situations in period 2 since two scenarios are created by each scenario from the previous period. Additionally, the third period has eight scenarios. There are a total of 15 nodes, numbered from 1 to 15. The nodes are shown in Figure 1.

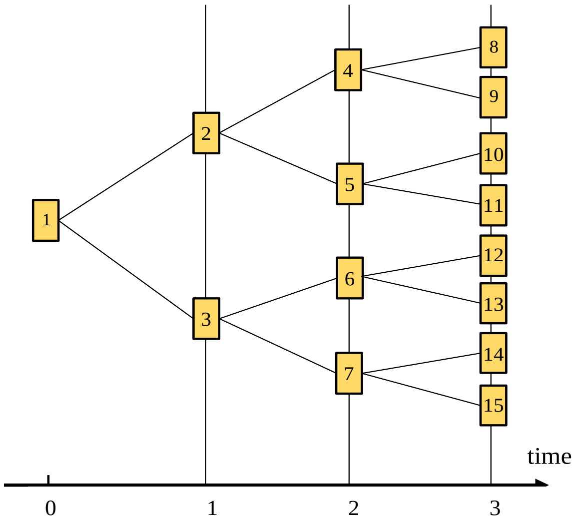

Figure 1. A scenario tree with 3 stages.

Tables 1-2 show the data set for return, and Tables 3-4 show the data set for liquidity of 20 stocks for three periods under different scenarios. The parameters of the proposed model, including lb_{it}^{φ}, ub_{it}^{φ}, c_{it}^{φ}, μ, and W are set equal to 0, 3.0000E+7, 0.1%, 1.5%, and 1.0000E+8, respectively. Also, the ideal goal of three objectives, including terminal wealth, risk, and liquidity, is set equal to 1.5000E+8, 0, and 1.0000E+8, respectively. Also, there are three weights assigned for deviational variables of goal k, and are determined by the decision maker and are the same.

Table 1. Data set for the return.

stock	node 2	node 3	node 4	node 5	node 6	node 7	node 8
01	0.00306	-0.02991	0.05847	0.09111	-0.13353	0.09457	0.15849
02	-0.08950	0.11736	0.07342	-0.11550	0.01835	0.04508	-0.11683
03	0.04082	0.04699	0.02386	-0.01969	0.02595	0.13615	0.02744
04	-0.12161	-0.05215	-0.11614	-0.00019	-0.03198	-0.13483	0.04704
05	0.05909	-0.02242	-0.0419	-0.16964	0.04657	-0.09378	0.13374
06	0.01498	0.07342	-0.02649	0.00754	-0.05872	0.08982	0.00603
07	0.04682	-0.12182	0.00619	-0.09047	0.01522	-0.0674	0.01438
08	-0.00187	-0.05769	0.05028	0.06628	-0.04433	0.10793	-0.00926
09	0.01334	0.11896	-0.14673	-0.10609	0.04298	0.05572	0.1465
10	0.06337	-0.0468	-0.02823	-0.05451	-0.01649	-0.14111	-0.11377
11	-0.05152	-0.13729	-0.05679	0.08499	0.01546	0.15781	-0.04892
12	-0.04635	-0.08068	-0.03017	-0.13756	0.01823	0.01431	-0.01203
13	-0.04445	-0.00591	0.12203	0.03697	0.04474	-0.10641	0.02864
14	-0.05018	-0.04778	-0.06655	0.03230	0.16687	-0.03262	-0.16677
15	-0.05944	0.09004	0.09816	-0.08714	0.07646	0.01540	0.04908
16	0.04687	-0.04014	0.10220	0.04578	-0.11788	0.15455	-0.04359
17	0.02099	0.01907	0.04576	0.00941	0.08696	0.09037	0.00047
18	-0.11949	-0.08947	-0.12438	-0.07118	-0.13244	0.18532	-0.08422
19	-0.16784	-0.05202	-0.05153	-0.00356	0.04542	0.01386	0.00641
20	0.15673	-0.01054	-0.02513	0.03650	0.01402	-0.12435	-0.05537

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Table 2. Data set for the return.

stock	node 9	node 10	node 11	node 12	node 13	node 14	node 15
01	-0.04711	-0.11384	0.10229	-0.05104	0.17628	0.11257	-0.04928
02	0.12718	-0.06329	-0.16137	-0.10303	-0.05822	0.03507	-0.02317
03	0.11938	-0.15234	-0.13755	-0.07847	0.0845	0.03729	0.06561
04	-0.12096	0.02555	-0.03724	0.05939	0.06459	-0.06936	-0.07786
05	0.09405	0.02783	-0.09798	-0.08576	-0.09141	0.10521	-0.01527
06	0.15651	0.09339	0.04491	0.13041	0.03973	0.03142	0.01044
07	0.02959	-0.03831	0.09162	0.06743	0.00862	-0.13501	-0.13108
08	-0.05608	-0.08498	-0.09715	-0.15106	-0.1174	0.0438	0.00529
09	-0.02407	-0.12139	-0.08019	-0.08225	0.04386	-0.10119	0.09118
10	0.01527	-0.06246	0.08364	0.07458	0.01145	0.02612	0.01235
11	0.13949	-0.03827	-0.06809	0.07507	0.16039	-0.04081	-0.10804
12	-0.12701	0.14469	0.07506	0.08674	0.03169	-0.16352	0.02684
13	-0.03174	-0.0237	0.09457	-0.02298	0.02442	0.0332	0.0279
14	0.06285	0.06484	-0.00955	-0.03201	0.11484	-0.00284	-0.02156
15	-0.00973	0.07402	0.0734	-0.01253	0.09891	0.03187	0.10655
16	-0.1078	-0.07023	0.00454	0.03847	-0.12382	0.14008	0.04978
17	-0.00176	-0.10114	-0.05543	0.06928	0.05049	-0.02558	0.1462
18	0.16892	0.05843	0.02963	-0.0044	0.1577	-0.08744	-0.04789
19	0.08036	0.09015	-0.12872	0.15236	-0.05981	0.02746	0.03159
20	-0.17557	0.08275	0.0675	0.05907	-0.13776	-0.07318	0.0648

Table 3. Data set for liquidity.

stock	node 2	node 3	node 4	node 5	node 6	node 7	node 8
01	0.06842	0.0525	0.02855	0.00805	0.02021	0.06172	0.01986
02	0.04595	0.0036	0.04774	0.02351	0.0159	0.01872	0.03084
03	0.09544	0.04411	0.03206	0.04246	0.00101	0.09877	0.09157
04	0.05561	0.00291	0.00284	0.00972	0.07923	0.05267	0.05143
05	0.00263	0.05748	0.08203	0.02578	0.07729	0.03694	0.06258
06	0.00126	0.06797	0.01666	0.03892	0.01464	0.0109	0.06306
07	0.03696	0.0704	0.06301	0.013	0.01321	0.07358	0.02697
08	0.06606	0.02243	0.07133	0.04477	0.07628	0.08613	0.04376
09	0.08392	0.0697	0.04599	0.02731	0.07816	0.08588	0.00218
10	0.06326	0.05019	0.02313	0.02248	0.05295	0.06571	0.08308
11	0.01312	0.06935	0.01072	0.0041	0.09274	0.04868	0.06616
12	0.09539	0.00514	0.05528	0.06905	0.01737	0.09225	0.09035
13	0.07638	0.0477	0.09292	0.06022	0.00128	0.01996	0.06487
14	0.02627	0.00487	0.01023	0.03446	0.07004	0.0809	0.09614
15	0.00258	0.00486	0.00093	0.01963	0.02847	0.03163	0.00645
16	0.0623	0.06253	0.06045	0.09468	0.04734	0.09704	0.06188
17	0.08472	0.08435	0.00575	0.02799	0.07336	0.07969	0.02672
18	0.04829	0.05965	0.03052	0.07917	0.07082	0.06821	0.07700
19	0.03952	0.09434	0.02895	0.04649	0.09938	0.00794	0.06952
20	0.07007	0.09706	0.02490	0.06680	0.04688	0.00446	0.03449

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Table 4. Data set for liquidity.

stock	scenario 2	scenario 3	scenario 4	scenario 5	scenario 6	scenario 7	scenario 8
01	0.00747	0.00059	0.09304	0.02300	0.01133	0.09857	0.05190
02	0.03484	0.01167	0.00477	0.04969	0.06179	0.08343	0.08904
03	0.09295	0.04833	0.09156	0.00174	0.02233	0.00069	0.09827
04	0.09769	0.08150	0.03714	0.05578	0.03701	0.07203	0.09779
05	0.08792	0.07238	0.04178	0.09838	0.06842	0.07682	0.08297
06	0.01583	0.09803	0.04995	0.02342	0.07503	0.05434	0.00477
07	0.06125	0.00380	0.04497	0.09935	0.02086	0.06457	0.09428
08	0.09997	0.07072	0.08116	0.08301	0.05001	0.08947	0.04364
09	0.04251	0.03974	0.04183	0.04432	0.02419	0.07534	0.09780
10	0.07671	0.08297	0.00348	0.08070	0.09370	0.05162	0.02949
11	0.06205	0.02146	0.00297	0.02561	0.08571	0.06426	0.03637
12	0.03151	0.04868	0.05538	0.02461	0.01195	0.05685	0.09623
13	0.04993	0.06653	0.02774	0.02770	0.08666	0.00283	0.00981
14	0.02161	0.02439	0.08380	0.09588	0.08639	0.06986	0.03599
15	0.08954	0.09406	0.00655	0.05872	0.08743	0.09087	0.05024
16	0.07742	0.04264	0.07977	0.03891	0.07791	0.02663	0.06918
17	0.00557	0.06566	0.07418	0.06777	0.06759	0.03596	0.05129
18	0.06124	0.06027	0.07711	0.01452	0.08441	0.02350	0.06718
19	0.05784	0.07070	0.06008	0.09784	0.07247	0.09927	0.00416
20	0.00180	0.07152	0.09876	0.03826	0.00188	0.01757	0.07779

According to Tables 1-4 and relevant explanations, the proposed model is run by GAMS 24.7. The results show the selected stocks. In seven nodes, the portfolio is selected, from nodes 1 to 7. Figure 2 indicates the visual distribution of the portfolio in each node. The stocks should be bought in maximum constraint (shown by dark green), and light green means partial investment in a stock. Stocks 1, 16, 17, 19, and 20 are robust. These stocks were selected more than twice. Compared with these stocks, 16, 17, 19 are more robust than 01 and 20, because of their fixed quantity. These stocks have the lowest transaction costs than others. In Table 6, the amount of each stock are indicated. Four stocks are selected in each node. Table 5 shows the optimal portfolio in each node.

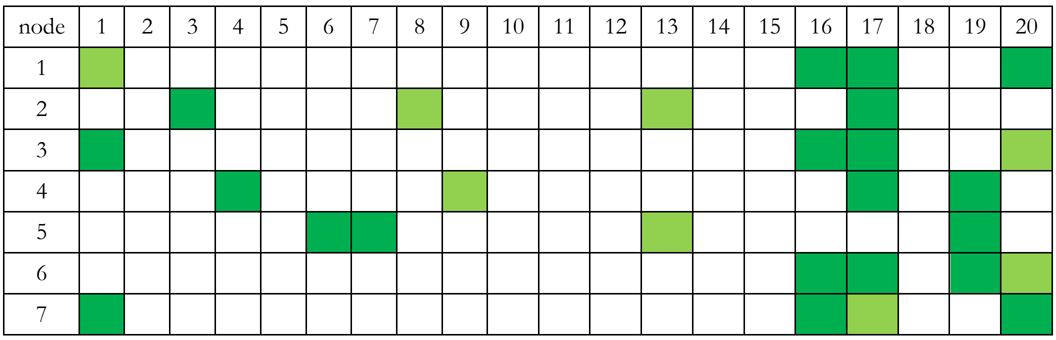

Figure 2. Distribution of portfolio in each node.

Table 5. Optimal portfolios for initial nodes at each period.

node	optimal portfolio
1	$(x_1 = 9.9001E+6,\ \ x_{16} = 3.0000E+7,\ \ x_{17} = 3.0000E+7\ \ x_{20} = 3.0000E+7)$
2	$(x_3 = 3.0000E+7,\ \ x_8 = 2.7947E+7,\ \ x_{13} = 1.8575E+7\ \ x_{17} = 3.0000E+7)$
3	$(x_1 = 3.0000E+7,\ \ x_{16} = 3.0000E+7,\ \ x_{17} = 3.0000E+7\ \ x_{20} = 8.6142E+6)$
4	$(x_4 = 3.0000E+7,\ \ x_9 = 2.2123E+7,\ \ x_{17} = 3.0000E+7\ \ x_{19} = 3.0000E+7)$
5	$(x_6 = 3.0000E+7,\ \ x_7 = 3.0000E+7,\ \ x_{13} = 1.8575E+7\ \ x_{19} = 3.0000E+7)$
6	$(x_{16} = 3.0000E+7,\ \ x_{17} = 3.0000E+7,\ \ x_{19} = 3.0000E+7\ \ x_{20} = 3.7367E+6)$
7	$(x_1 = 3.0000E+7,\ \ x_{16} = 3.0000E+7,\ \ x_{17} = 1.7694E+7\ \ x_{20} = 3.0000E+7)$

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Figure 3 shows the value of functions in each node through investment horizon, return, risk, and liquidity for each portfolio. In nodes 3, 6, and 13, the return is negative, and there is a significant difference between them. Also, they are riskier than others. In nodes 11 and 12, although returns are positive, risks are more than returns. The highest return is obtained in Node 8. Also, Node 11 is the riskiest portfolio, and Node 7 is the lowest. There is a hidden risk because liquidity has a significant difference between return, and risk in some nodes such as 8, 9, and 11. However, optimal obtained return, risk, and liquidity are 1.097483E+8, 2.176583E+7 and 4.699963E+7, respectively. Also, the transaction cost is 722,969 units. So, it is better for the investor to keep the investment to obtain gain.

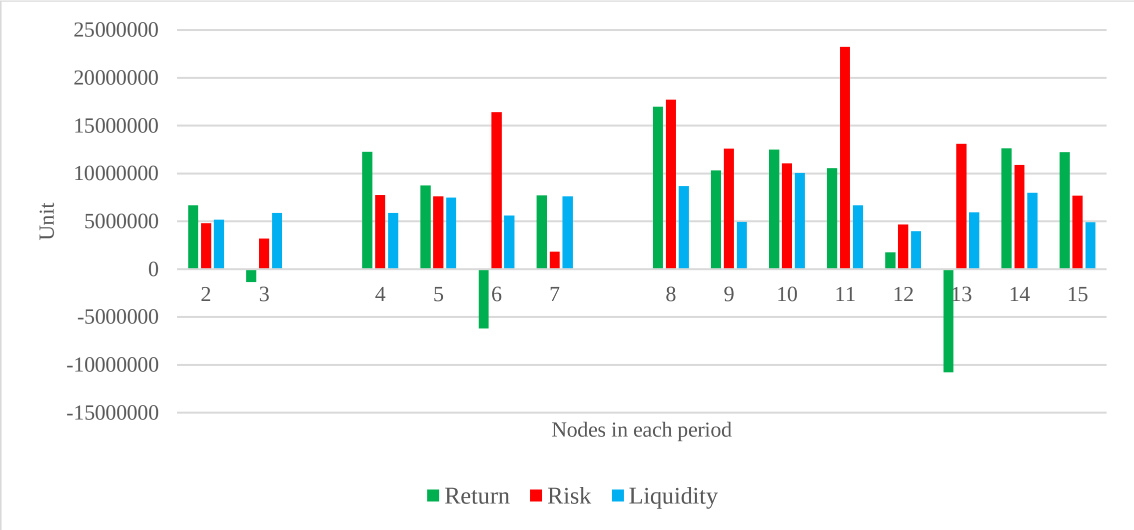

Figure 3. Function value in each node.

4.1. Managerial insights

The suggested model in this work has three goals: the first goal is wealth, the second goal is to evaluate downside risk using semi-deviation, and the third goal is to quantify portfolio liquidity. The suggested linear model offers a comprehensive solution. Additionally, the transaction cost is considered, leading to the proposal of a novel viewpoint on multi-stage stochastic situations. The goal programming (GP) approach is then used to find the best action. The suggested stochastic multi-objective model is reduced to a clear single-objective issue using a scenario tree technique and GP.

Transaction charges apply to the purchasing and selling shares on the stock market. Excessive buying and selling of stocks with slight improvement is avoided, and stocks that are steady at all or most times during the investment period are chosen by integrating transaction costs in the model, making it more accurate. Also, to avoid having issues converting their stocks into cash at the conclusion of the investment term, investors may pick equities in their portfolio with a high sales speed by taking the liquidity factor into account in the suggested model.

Each of the outlined situations has a possibility of occurring in an unpredictable environment. The suggested model provides an ideal solution and makes profits regardless of how each scenario plays out. The investor is required to hold the investment until it matures. Since the best answer takes all periods into account, the ultimate result has the possible profit. There is a chance of losing money if the investor sells the investment during any of the transitional periods (from period 2 to t-1). He will lose 6198448 units if he sells his investment in the second period after Node 6 has occurred.

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4.2. The expected value of perfect information (EVPI) and value of the stochastic solution (VSS)

A mean-value deterministic multi-period portfolio model is suggested to indicate the originality of this study and to show why the multi-stage stochastic portfolio method is preferable. WS stands for "wait and see,"; thus, the decision-maker must hold off choosing until all available information has been provided. The expected value (EV) solution results from the deterministic model with an average yield. The specific objective values for each scenario may be derived using the EV solution. The expected value, or the anticipated outcome of employing the EV solution, is then obtained by multiplying these objective values by the chance that the associated scenario will occur (EEV). The "here and now" variant of stochastic programming, often known as SP, represents its maximized profit value. The anticipated value of perfect information (EVPI) and the value of the stochastic solution (VSS) are two indicators that are employed in the study.

The expected value of perfect information (EVPI) is computed to quantify the impact of uncertainty on decisions. This metric calculates the anticipated profit loss. Better projections will not be helpful if the EVPI is highly low, and inadequate future knowledge might be expensive if it is somewhat large.

In addition, VSS is used to gauge the stochastic programming model's capacity and to boost profits. When parameters are fixed to average values, and the corresponding optimal solution is used, it is the difference between the solution of the SP model and the predicted value of the objective function. VSS illustrates how much more we can obtain if SP is applied. The SP result is better when VSS is higher than the projected result.

Table 6 displays the pertinent metrics for the case problem in this study. Table 6 shows that SP makes more money than EV. The positive VSS values show the importance of letting the amount allotted to each stock be changed for various situations at each stage of the decision-making process rather than setting its value at the beginning of the planning horizon. Thus, the shortcomings of the conventional deterministic model can be improved by the multi-stage approach described in this work.

Table 6. The related measurements for the case problem.

Measurements	Return	Liquidity
(1) WS	1.10448E+08	7.80970E+07
(2) SP	1.09748E+8	4.69996E+7
(3) EV	6.75278E+7	1.06308E+7
(4) EVPI=WS-SP	7.00000E+5	3.10974E+7
(5) VSS=SP-EV	4.22202E+7	3.63688E+7
(6) (VSS/EV)*100%	62%	342%

5. Conclusions and future research directions

Stochastic programming models are flexible ways in optimizing financial problems under uncertainty. In the prior literature, numerous formulations for the multi-stage financial problem have been proposed (Kall, Wallace, & Kall, 1994). The asset/liability management problem was built by Carino et al. (1994) using a multi-stage stochastic programming model. A hybrid simulation/tree multi-period stochastic programming model has been created for optimal asset distribution (Hibiki, 2006). Scenarios are frequently used to simulate random parameters in multi-period stochastic programming models. Scenarios are constructed using a tree structure (Mulvey & Ziemba, 1995). The model, which is based on the extension of the decision space, takes into consideration the conditional nature of the scenario tree.

Researchers frequently employed variation as a risk indicator. Downside risk indicators like semi-deviation should be replaced with variance since they better reflect actual investment risk, asymmetric return distributions, and investor needs. The semi-deviation method, as a risk management approach, describes the loss happening over a certain period at a specific confidence level (Ji and Lejeune, 2018). Although the previous studies have considered the multi-stage stochastic portfolio selection issue, the majority of these studies have only employed one objective function using the mean or risk of the portfolio. Specifically, stock liquidity has not been taken into account as an objective function. Additionally, most of these studies are nonlinear, and they do not offer a global solution, and accordingly, investors cannot trust such a portfolio.

The suggested model in this work has three goals: the first goal is wealth, the second goal is to evaluate downside risk using semi-deviation, and the third goal is to quantify portfolio liquidity. The suggested linear model offers a comprehensive solution. Additionally, the transaction cost is considered, leading to the proposal of a novel viewpoint on multi-stage stochastic situations. The goal programming (GP) approach is then used to find the best action. The suggested stochastic multi-objective model is reduced to a single objective issue using GP and a scenario tree technique. Also, the proposed portfolio strategy allows a fair trade-off among revenue, risk, and liquidity goals. Providing a real case study, the efficacy of the constructed portfolio optimization model and the value of the suggested solution strategy are examined. In the case study, there are 14 nodes in the tree structure, creating 8 situations throughout 3 periods. Calculated variables like VSS and EVPI demonstrate the robustness of the suggested model.

In multi-period portfolio selection, this study introduces important issues and new lines of inquiry. As a result, we plan to further this study in the following areas:

Cardinality. The cardinality requirement wasn't used in this study to keep things mathematically simple. However, it is advised to employ cardinality to increase investor control over the portfolio.

Mixed-type uncertainty. The scenario tree idea was used to model. For future studies, it is advised to extend this proposed model to account for hybrid uncertainties, such as interval variables and scenario trees.

Portfolio selection. Portfolio optimization might be preceded by portfolio selection. This step helps choose suitable stocks for investment. One of the techniques used for selecting a portfolio before the optimization is data envelopment analysis (Nouri et al., 2019).

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